

How to Decide with PROMETHEE

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Introduction

Most of the economical, industrial, financial or political decision problems are multicriteria. Nobody buys a car on base of the price only (financial criterion); the comfort, the quality, the performance, the prestige, ... is obviously always taken into account. On the other hand nobody reacts in the same way. Indeed one can observe many different cars in the streets. The selection is submitted to each individual's personal taste. Everybody allocates a different set of weights to the criteria.

The problem of the selection or the ranking of alternatives submitted to a multicriteria evaluation is not an easy problem. Neither economically nor mathematically! Usually there is no optimal solution; no alternative is the best one on each criterion. A better quality implies a higher price. The criteria are conflicting. Compromise solutions have to be considered. Why not the best compromises?

In the recent years several *decision aid* methods or *decision support systems* have been proposed to help in the selection of the best compromise alternatives. In this paper we give, on basis of a short example, an overview of the PROMETHEE-GAIA methodology for treating multicriteria problems. This methodology is known as one of the most efficient but also one of the easiest in the field. Particularly user-friendly software, called *Decision Lab* has been developed in collaboration with the Canadian company *Visual Decision* to assist all kinds of decision-makers. This software treats matrices similar to that of Table 1, where $a_1, a_2, \dots, a_i, \dots, a_n$ are n potential alternatives and $f_1, f_2, \dots, f_j, \dots, f_k$ are k evaluation criteria. Each evaluation

$f_j(a_i)$ must be a real number. Such a matrix can model many real-world applications. In some cases it is an easy task and the matrix is obtained immediately. In other cases it can be a hard problem implying several months of severe consultancy and analysis work, as for instance when a new production unit must be selected among several possible sites.

Several authors such as B. Roy (1985), D. Bouyssou (1984), R. Keeney (1992), S. Zionts (1989) and P. Vincke (1992) have analysed how to model a real-world multicriteria situation. Their contributions are impressive. They stress out that the matrix often must be evolutive : additional alternatives may be considered according to the gain in information during the progress of the decision procedure. New evaluation criteria could be considered, some others temporarily deleted. The structuration of the matrix is achieved progressively. Normative, constructive, descriptive and prescriptive arguments can be considered. Whereas the *Decision Lab* software provides ways to handle easily evolutive matrices, the PROMETHEE methodology doesn't take into account such arguments by itself. It starts when the matrix is available. Even in this case it is not an easy task to finalize the decision. Where

	$f_1(\cdot)$	$f_2(\cdot)$	$f_j(\cdot)$	$f_k(\cdot)$
a_1	$f_j(a_i)$					
a_2						
...						
a_i						
...						
a_n						

- Table 1 -

are the best compromises? What is the best ranking?

Requisites can be formulated for an appropriate multicriteria procedure. It must be flexible. A thorough sensitivity analysis must be possible. Indifference or gradual degrees of preference have to be associated to the deviations observed between the evaluations. Partial rankings (including the natural avowal of incomparable alternatives) and complete rankings (without) should be obtained. A clear view of the conflicting character of the criteria and the influence of a particular set of weights should be displayed. It should also be possible to consider additional limitation constraints; all these requisites are taken into account by the PROMETHEE-GAIA procedure.

An example

Let us suppose that a bicycle manufacturing company is intending to advertise its products. After long discussions, six marketing actions are considered: advertising in the international newspaper *News*, in the newspaper *Herald*, by mean of advertising boards in large cities, of a personal mailing, by TV spots on channels *CMM* or *NCB*. As one can see by looking at the enumeration of the advertising supports, this is an illustrative example, not a real-world one! As mentioned in Table 2, five evaluation criteria are taken into account: the cost expressed in 1,000 US\$, the target expressed in 10,000 people, the duration of the action in days, the efficiency expressed on a 0-100 scale and the number of people involved in the action within the company (manpower). The matrix includes 30 evaluations. Some criteria have to be minimized (cost and manpower), the others have to be maximized (target, duration, efficiency). Equal weights have been initially allocated to all the criteria. Even for such a small problem, it is not obvious to detect the best advertising action. Indeed no alternative is optimal on the five criteria!

Criteria	C1	C2	C3	C4	C5
min/max	cost min	target max	durat. max	effic. max	manp. min
<i>News</i>	60	900	22	51	8
<i>Herald</i>	30	520	31	13	1
Panels	40	650	20	58	2
Mailing	92	750	60	36	3
<i>CMM</i>	52	780	58	90	1
<i>NCB</i>	80	920	4	75	6

- Table 2 -

The PROMETHEE & GAIA analysis¹

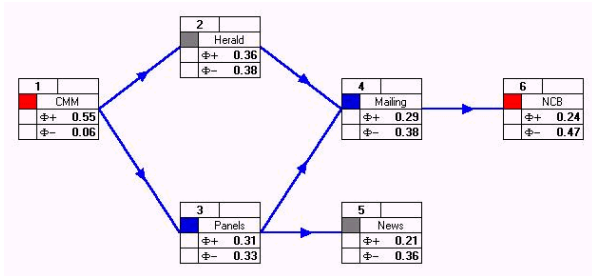
It is not the purpose of this paper to explain in detail the PROMETHEE methodology. See for instance Brans et al (1982, 1984, 1985, 1986, 1992, 1994) and Mareschal et al (1986, 1988). We will only comment on the results provided by the *Decision Lab* software on the above-mentioned example.

PROMETHEE requests additional information. For each criterion a specific preference function must be defined. This function is used to compute the degree of preference associated to the best action in case of pairwise comparisons. Six possible shapes of preference functions are available in the software. These are described for instance in Brans et al (1986). In this example, the shapes 5 (linear), 3 (V-shape), 2 (U-shape), 6 (Gaussian) and 1 (usual) have been respectively associated to the five criteria.

PROMETHEE & GAIA calculates positive and negative preference flows for each alternative. The positive flow is expressing how much an alternative is *dominating* (power) the other ones, and the negative flow how much it is *dominated* (weakness) by the other ones. Based on these flows, the PROMETHEE I partial ranking is obtained (Fig.1). One can immediately see that the *CMM* action dominates all the others. It looks as the best compromise, according to the basic weight distribution. On the other hand the two newspapers *News* and *Herald* are incomparable actions: the first one is a strong action (high

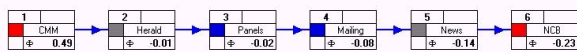
¹ PROMETHEE : Preference Ranking Organisation METHod for Enrichment Evaluations. GAIA : Geometrical Analysis for Interactive Assistance.

cost, high target, high efficiency) while the second is more limited. PROMETHEE I does not compare conflicting actions.



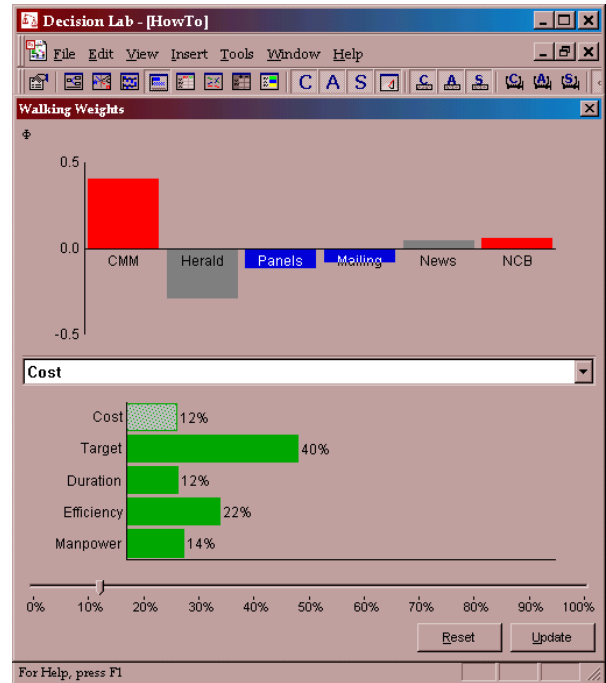
- Fig.1 : PROMETHEE I ranking -

On the other hand PROMETHEE II provides a complete ranking (Fig.2). It is based on the balance of the two preference flows. The information looks stronger but some part of it get lost in the process. Both PROMETHEE I and II help the decision-maker to finalize the selection of a best compromise. A clear view of the outranking relations between the alternatives is obtained.



- Fig.2 : PROMETHEE II ranking -

Obviously the PROMETHEE I and II rankings are influenced by the weights allocated to the criteria. A special feature of the software, called *The Walking Weights*, (Fig.3) allows to modify the weights and to observe the resulting modifications of the PROMETHEE II ranking. For the following weight distribution (12, 40, 12, 22, 14) one observes easily that CMM still dominates the other ones. Its position as a best compromise seems to be very stable. On the other hand, the ranking of the last five actions is now completely opposite. Such a sensitivity analysis tool is particularly valuable when the decision-maker has no predetermined weights in mind.



- Fig.3 : Walking Weights -

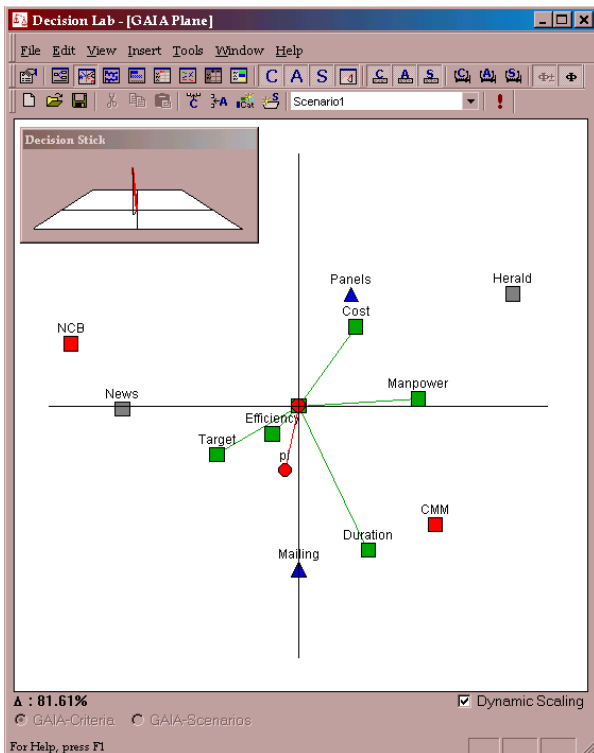
The information relative to a decision problem including k criteria can be represented in a k -dimensional space. The GAIA plane is obtained by projection of this information on a plane such that as few information as possible get lost. Alternatives are represented by points and, criteria by axes. The conflicting character of the criteria appears clearly (Fig.4): criteria expressing similar preferences on the data are oriented in the same direction, conflicting criteria are pointing in opposite directions. In this case we observe for instance that the cost is in strong conflict with the target. It is also possible to appreciate clearly the quality of the alternatives with respect to the different criteria. A1 (*News*) and A6 (*NCB*) are particularly good on C2 (*Target*) and C4 (*Efficiency*). A5 (*CMM*) and A4 (*Pers.Mailing*) look good on C3 (*Duration*), ...

In addition to the representation of the alternatives and criteria, the projection of the weights vector in the GAIA plane corresponds to another axis (π , the PROMETHEE decision axis) that shows the direction of the compromise resulting from the weights allocated to the criteria. The decision-maker is thus invited to consider the alternatives located in that direction. In the case of equal weights, it is confirmed that A5 (CMM) is the best compromise. When the weights are modified, the positions of the alternatives and the criteria remain the same, only the decision axis π is changing. The software allows using the weights vector as a *decision stick* to orientate the decision. The movements of the stick corresponding to modifications of the weights are directly displayed in the 3D-view window of the GAIA screen (Fig.4). Decision-makers particularly appreciate this sensitivity analysis tool.

When the decision-maker is not able or does not want to allocate precise weights to the criteria, it is possible to specify intervals of possible values rather than one fixed value for each weight. In this case, the PROMETHEE VI

procedure can be used to indicate whether the problem is *soft* or *hard*. It is *soft* when the decision axis π always remains in the same general direction for the weight distributions that are compatible with the intervals. It is *hard* when opposite directions are possible depending on the actual values of the weights. In the case of a *hard* problem, the decision-maker should concentrate on more precise values of the weights. This feature is currently not implemented in *Decision Lab*.

It is often the case that several alternatives have to be selected and that additional constraints must hold. In this case a (0-1) linear program can be associated to the problem, the solution of which gives the best selection of r actions among n . Let us suppose for instance that the decision-maker is wishing to undertake at least two advertising actions and at most four, that he wants to advertise in exactly one newspaper and exactly on one TV channel, that a minimum expected return of 1200 (US\$ 1000) must be reached (expected returns have been estimated for each alternative) and that he doesn't want that the cost of advertising in newspapers be higher than 50% of the total cost. Then the following (0-1) LP is considered:



- Fig.4 : GAIA -

$$\text{Max } \phi(A1) x_1 + \dots + \phi(A6) x_6$$

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \geq 2$$

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \leq 4$$

$$x_1 + x_2 = 1$$

$$x_5 + x_6 = 1$$

$$625 x_1 + 550 x_2 + 250 x_3 + 150 x_4 + 175 x_5$$

$$+ 750 x_6 \geq 1200$$

$$- 60 x_1 - 30 x_2 + 40 x_3 + 92 x_4 + 52 x_5$$

$$+ 80 x_6 \geq 0$$

Where $x_i = 1$ if A_i is selected and 0 otherwise.

The objective function expresses that as much outranking flow $\phi(\cdot)$ as possible should be collected on the set of selected alternatives.

The PROMETHEE V procedure allows investigating such problems. In this case, the optimal solution is the following:

A1: Herald, A3: Boards, A4: Mailing,
A5 : CMM

The *Decision Lab* software

The Decision Lab software is an up-to-date implementation of the PROMETHEE & GAIA methods. It includes many practical developments, such as the treatment of missing values, the definition of categories of actions or criteria, as well as powerful group decision extensions through the definition of multiple scenarios. The Canadian company Visual Decision develops decision Lab. It works under Windows 95, 98, NT or 2000 on PC compatible microcomputers. A demo version as well as full versions (Executive or Developer – including programming capabilities) is available from Visual Decision.

Visit <http://www.visualdecision.com>

Further readings

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